विद्या सर्वार्थ साधिका

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MID TERM EXAMINATION
Class : XII

Subject: Mathematics M.M: 80
Date : 23/09/2019 Time: 3 Hours

General Instructions:

i) All questions are compulsory.
ii) This question paper contains 36 questions.
iii) Questions 1- 20 in Section A are very short-answer type questions carrying 1 mark each.
iv) Questions 21 - 26 in Section B are short-answer type questions carrying 2 marks each.
v) Questions 27 - 32 in Section C are long-answer – I type questions carrying 4 marks each.
vi) Questions 33 - 36 in Section D is long-answer – II type questions carrying 6 marks.

SECTION-A

1. The roots of the equation $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$ are (1)

(a) 1, 2 (b) - 1, 2 (c) 1, - 2 (d) - 1, - 2

2. The value of $\cot^{-1} 9 + \operatorname{cosec}^{-1} \frac{\sqrt{41}}{4}$ *is*_____ (1)

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) π

3. $\int_0^{\pi} \frac{dx}{1+\sin x} =$ _____ . (1)

(a) 0 (b) ½ (c) 2 (d) 3/2

4. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, then $(A^2 - 5A)\, A^{-1}$ = _____ (1)

(a) $\begin{bmatrix} 4 & 2 & 3 \\ -1 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 2 & 3 \\ -1 & -4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -4 & -1 & 1 \\ 2 & -4 & 2 \\ 3 & 2 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -2 & 1 \\ 4 & -2 & -3 \\ 1 & 4 & -2 \end{bmatrix}$

5. Let $f(x) = x^3$ and $g(x) = 3^x$. The values of a such that $g(f(a)) = f(g(a))$ are (1)

(a) 0, 2 (b) 1, 3 (c) 0, $\pm\sqrt{3}$ (d) 1, $\pm$ 2

6. $\int x^{51} (\tan^{-1} x + \cot^{-1} x) dx =$ _____. (1)

(a) $\frac{x^{52}}{52} (\tan^{-1} x + \cot^{-1} x) + c$ (b) $\frac{x^{52}}{52} (\tan^{-1} x - \cot^{-1} x) + c$

(c) $\frac{\pi\, x^{52}}{104} + \frac{\pi}{2} + c$ (d) $\frac{x^{52}}{52} + \frac{\pi}{2} + c$

7. If $u = e^{\sin^{-1} x}, v = \log x$ then $\frac{du}{dv} =$ _____ (1)

(a) xu (b) $\frac{u}{\sqrt{1-x^2}}$ (c) $\frac{ux}{\sqrt{1-x^2}}$ (d) None of these

8. $\int \frac{dx}{(1+e^x)(1+e^{-x})} =$ (1)

(a) $\frac{-1}{1+e^x}$ (b) $\frac{e^x}{1+e^x}$ (c) $\frac{1}{1+e^x}$ (d) none of these

9. In the following question, Statement – 1 is followed by Statement – 2. Mark the correct choice as: (1)

Statement – 1 : $\int \frac{1}{4e^{-x}-9e^x} dx = \frac{1}{6} \log \left|\frac{2+3e^x}{2-3e^x}\right| + c$

Statement – 2 : $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \log \left|\frac{a+x}{a-x}\right| + c$

(a) Statement – 1 is true, Statement – 2 is true, Statement – 2 is a correct explanation for Statement- 1.
(b) Statement – 1 is true, Statement – 2 is true, Statement – 2 is not a correct explanation for Statement- 1.
(c) Statement – 1 is true, Statement – 2 is false.
(d) Statement – 1 is false, Statement – 2 is true.

10. In the following question, Statement – 1 is followed by Statement – 2. Mark the correct choice as: (1)

Statement – 1 : The function $f(x) = x^3 - 3x^2 + 12x$ is increasing on R.

Statement – 2 : If a differentiable function g(x) is increasing implies $g'(x) > 0$.

(a) Statement – 1 is true, Statement – 2 is true, Statement – 2 is a correct explanation for Statement- 1.
(b) Statement – 1 is true, Statement – 2 is true, Statement – 2 is not a correct explanation for Statement- 1.
(c) Statement – 1 is true, Statement – 2 is false.
(d) Statement – 1 is false, Statement – 2 is true.

11. Let N be the set of natural numbers and relation R on N be defined by (1)
R = {(x, y): x, y ∈ N, x + 4y = 10}. Determine whether the above relation is reflexive, symmetric.

12. If $y = 9^{\log_3 x}$, show that $\frac{dy}{dx} = 2x$. (1)

13. If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$ then find $\cot^{-1} x + \cot^{-1} y$. (1)

14. Evaluate: $\sin^{-1}(\sin(-600°))$. (1)

15. If $y = e^{x+e^{x+e^{x+\cdots..\infty}}}$,prove that $\frac{dy}{dx} = \frac{y}{1-y}$. (1)

16. Find the integral value(s) of x if $\begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix} = 28$. (1)

17. If x changes from 4 to 4.01, then find the approximate change in $\log_e x$. (1)

18. Let *f*: R → R be defined as $f(x) = x^2 + 1$. Find the value of $f^{-1}\{37\}$. (1)

19. Find the value of x and y, given that $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$. (1)

20. Differentiate $sin^2 x$ with respect to $e^{\cos x}$. (1)

SECTION-B

21. If $f(x) = \sqrt{x^2 + 1}; g(x) = \frac{x+1}{x^2+1}$ *and* $h(x) = 2x - 3$,then find $f'[h'\{g'(x)\}]$. (2)

22. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find the value of x and y such that $A^2 + xI_2 = yA$. (2)

OR

If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, find a and b.

23. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to x – axis. (2)

24. If $y = (x + \sqrt{1 + x^2})^n$, then show that $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y$. (2)

OR

Differentiate the function $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ with respect to x.

25. Find the maximum and minimum values of the function *f* given by $f(x) = \sin x + \cos x$. (2)

OR

Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly increasing or decreasing. Also find the points on which the tangents are parallel to x – axis.

26. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{sec^2 x}{(1+\tan x)(2+\tan x)}\, dx.$ (2)

SECTION – C

27. Let $A = \begin{bmatrix} 1 & -2 \\ 5 & 4 \\ 3 & 0 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ -3 & 5 \end{bmatrix}$ *and* $C = \begin{bmatrix} 4 & 3 \\ -2 & 2 \\ 1 & 6 \end{bmatrix}$. Verify that (A +B) + C = A + (B + C). (4)

28. Prove that: $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2.$ (4)

OR

Prove that $\tan^{-1}\frac{yz}{xr} + \tan^{-1}\frac{zx}{yr} + \tan^{-1}\frac{xy}{zr} = \frac{\pi}{2}$, where x, y, z > 0 such that $x^2 + y^2 + z^2 = r^2$.

29. Verify Langange's mean value theorem for $f(x) = \sqrt{x^2 - x}$ *in* $[1, 4]$. (4)

30. By using properties of determinants, prove that : (4)

$$\begin{vmatrix} 1 + sin^2x & cos^2x & 4\sin 2x \\ sin^2x & 1 + cos^2x & 4\sin 2x \\ sin^2x & cos^2x & 1 + 4\sin 2x \end{vmatrix} = 2 + 4\sin 2x.$$

OR

By using properties of determinants, prove that :

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ba & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix} = a^2 + b^2 + c^2 + 1.$$

31. Prove that $\sin x(1 + \cos x)$ has a maximum value for $x = \frac{\pi}{3}$. (4)

32. Evaluate: $\int_0^1 \frac{x\,e^x}{(x+1)^2}\,dx$. (4)

OR

Evaluate: $\int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}}dx$.

SECTION-D

33. Let N be the set of all natural numbers and let R be a relation in N, defined by (6)
R = {(a, b): a is a multiple of b} . Show that R is reflexive and transitive but not symmetric.

OR

Let $A = R - \left\{\frac{3}{5}\right\}$ *and* $B = R - \left\{\frac{7}{5}\right\}$ let *f*: A → B: $f(x) = \frac{7x+4}{5x-3}$ and *g*: B → A: $g(y) = \frac{3y+4}{5y-4}$.
Show that *(gof) = I_A and (fog) = I_B.*

34. Evaluate: $\int_1^3 (2x^2 + 3)dx$ as limit of sums. (6)

35. Find $A^{-1}, where\ A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$. Hence solve the system of equations: (6)
4x + 2y + 3z = 2, x + y + z = 1, 3x + y – 2z = 5.

OR

If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}, find\ (A')^{-1}$.

36. Show that the semi – vertical angle of a cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$. (6)